
**EMISSION MEASUREMENT CENTER
TECHNICAL INFORMATION DOCUMENT**

DERIVATION OF $\Delta H_{@}$

INTRODUCTION

The term $\Delta H_{@}$ given in Equation 1 below is determined during the calibration of the orifice meter in the Environmental Protection Agency (EPA) Method 5 sampling train. The purpose of this document is to explain the derivation of this term.

$$\Delta H_{@} = \frac{0.0319 \Delta H}{P_b(t_o + 460)} \left[\frac{(t_w + 460)\theta}{V_w} \right]^2 \quad Eq. 1$$

DERIVATION

The flow rate through an orifice meter is defined by the following equation:

$$Q_m = K_m \sqrt{\frac{T_m \Delta H}{P_m M_m}} \quad Eq. 2$$

Where:

Q_m = volumetric flow rate through the orifice meter, cfm.

K_m = orifice meter calibration constant, [(in. Hg)(lb-mole)/[(°R)(in. H₂O)]].

T_m = absolute orifice meter temperature, °R.

ΔH = Orifice meter pressure differential, in. H₂O.

P_m = absolute office meter pressure, in Hg.

M_m = molecular weight of gas flowing through the orifice meter, lb/lb-mole.

Instead of using the orifice meter coefficient K_m , the term $\Delta H_{@}$ was derived to reflect the general sampling conditions of sampling trains and orifice meters designs with a calibration constant that could be related to the desired flow rate. For example, a $\Delta H_{@}$ of 1.84 means that the orifice pressure differential ΔH would be about 1.84 when sample gas is flowing through the orifice meter. The reference conditions for ΔH were chosen to be 0.75 dcfm air at 68°F and 29.92 in. Hg. Using Equation 2, ΔH is then defined as:

$$\Delta H_{@} = \frac{Q_{@}^2 P_{@} M_{@}}{K_m^2 T_{@}} \quad Eq. 3$$

Where the subscript “@” represents the reference conditions of 0.75 dcfm of air at 68°F and 29.92 in. Hg.

Substituting the values for the reference conditions (except $M_{@}$) into Equation 3, the following is obtained:

$$\Delta H_{@} = \frac{(0.75^2)(29.92)M_{@}}{528 K_m^2} = 0.0319 \frac{M_{@}}{K_m^2} \quad Eq. 4$$

Now, the relationship between Q_m and the volumetric flow rate through the wet test meter (Q_w) is as follows (the subscript “w” refers to the flow rate conditions at the wet test meter):

$$\frac{P_m Q_m}{T_m} = \frac{P_w Q_w}{T_w} \quad Eq. 5$$

Substituting Equation 2 into Equation 5 and solving for K_m^2 :

$$\frac{P_m K_m}{T_m} \sqrt{\frac{T_m \Delta H}{P_m M_m}} = \frac{P_w Q_w}{T_w}$$

$$K_m^2 = \frac{M_m T_m P_w^2 Q_w^2}{\Delta H T_w^2 P_m} \quad Eq. 6$$

In the Method 5 calibration set-up, the pressure at the orifice meter and the wet test meter are at barometric pressure. Thus, $P_w = P_m = P_b$, Equation 6 simplifies to:

$$K_m^2 = \frac{M_m T_m P_b Q_w^2}{\Delta H T_w^2} \quad Eq. 7$$

Substituting Equation 7 into Equation 4 :

$$\Delta H_{@} = \frac{0.0319 M_{@} \Delta H T_w^2}{M_m T_m P_b Q_w^2} \quad Eq. 8$$

Since air is being used to calibrate the orifice, $M_{@} = M_m$. In addition, $Q_w = V_w/\theta$, which is the volume measured by the wet test meter divided by time, and T_m is the same temperature

measured at the outlet of the dry gas meter, i.e., $T_m = T_o$. Furthermore, $T_w = t_w + 460$, and $T_o = t_o + 460$. Making these substitutions in Equation 8 yields:

$$\Delta H_{@} = \frac{0.0319 \Delta H}{P_b(t_o + 460)} \left[\frac{(t_w + 460)\theta}{V_w} \right]^2 \quad Eq. 1$$

Where:

$\Delta H_{@}$ = ΔH that gives 0.75 dcfm of air at 68°F and 29.92 in. Hg, in. H₂O.

0.0319 = constant, $(0.75^2)(29.92)/(68 + 460)$.

ΔH = orifice meter pressure differential, in. H₂O.

t_w = wet test meter temperature, °F.

θ = time, min.

P_b = barometric pressure, in. Hg.

t_o = outlet temperature of dry gas meter, °F.

V_m = volume measured by the wet test meter, cf.

BIBLIOGRAPY

1. Code of Federal Regulations, Title 40, Part 60. U.S. Government Printing Office, Washington, D.C. July 1, 1987
2. Shigehara, R.T., Adjustments in the EPA Nomograph for Different Pitot Tube Coefficients and Dry Molecular Weights. In: Stack Sampling Technical Information – A Collection of Monographs and Papers, Vol. III. U.S. Environmental Protection Agency. Publication No. EPA-450-2-78-042c. October 1978. pp. 48-49.
3. Smith, W.S., R.M. Martin, D.E. Durst, R.G. Hyland, T.J Logan, and C.B. Hager. Stack Gas Sampling Improved and Simplified with New Equipment. Presented at the 60th Annual Meeting of the Air Pollution Control Association, June 11-16, 1967, Cleveland, Ohio. APCA Paper N. 67-119.