Hierarchical Techniques

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Bayesian Modeling

*** Unknown = “Random”

*** Learn/update from data

*** Prior information

*** Uncertainty management!!!

Bayesian Hierarchical Modeling BHM

Why?

*** Dealing with complexity

*** Combining information:

Diverse datasets & Science

How?

Sequence of probability models

BHM Skeleton:

1. [data | process, $\boldsymbol{\theta}$]
2. [process | $\boldsymbol{\theta}$]
3. [$\boldsymbol{\theta}$]

Posterior: via Bayes' Theorem

4. [process, $\boldsymbol{\theta}$ | data]

1. Data Model

* Conditional distribution of data
* Combine diverse datasets

2. & 3. Priors

* Physical modeling

4. Posterior

* Combination, with uncertainties

Ozone Prediction

$\boxed{\text{Processes}}$

Ozone & Meteorology: O(s,t) & M(s,t)

Fields in space s & time t

$\boxed{\text{Station Data}}$

***Sampling–Support Issues

***Met. & Ozone data not co-located)

$\boxed{\text{Science}}$

***Atmos: Dynamics & Chem.

***Sources

***Representation via Comp. Models

BHM Skeleton

1. $[\, D_{\mathbf{O}}, D_{\mathbf{M}} \mid O(s,t), M(s,t), \boldsymbol{\theta} \,]$
2. $[\, O(s,t), M(s,t) \mid \boldsymbol{\theta}]$
3. $[\, \boldsymbol{\theta} \,]$

Keys

1. $[\, D_{\mathbf{O}}, D_{\mathbf{M}} \mid O(s,t), M(s,t), \boldsymbol{\theta} \,]$

$= [\, D_{\mathbf{O}} \mid O(s,t), \boldsymbol{\theta} \,]\, [\, D_{\mathbf{M}} \mid M(s,t), \boldsymbol{\theta} \,]$

2. $[\, O(s,t), M(s,t) \mid \boldsymbol{\theta}]$

$= [\, O(s,t) \mid M(s,t), \boldsymbol{\theta}]\, [\, M(s,t) \mid \boldsymbol{\theta}]$

Example: “Chicago” Plus

10 by 10 Grid.

$O(\cdot,t)$ = 100-dimensional vector. t=day.

Focus: Ozone, treat Met as given.

$\boxed{\text{Stages}}$

1. $[D(\cdot,1),...,D(\cdot,T) \mid O(\cdot,0),O(\cdot,1),...,O(\cdot,T),M,\boldsymbol{\theta}]$

$= \text{product } [D(\cdot,t) \mid O(\cdot,t),M,\boldsymbol{\theta}]$

2. $[O(\cdot,0),O(\cdot,1),...,O(\cdot,T) \mid M,\boldsymbol{\theta}]$

$= [O(\cdot,0) \mid M,\boldsymbol{\theta}] \text{ product } [O(\cdot,t) \mid O(\cdot,t\text{-}1)\ M,\boldsymbol{\theta}]$

3. $[\boldsymbol{\theta}]$

$\boxed{\text{Alternate Representation}}$

1. $D(\cdot,t) = K\ O(\cdot,t) + e(\cdot,t)$

$e(\cdot,t)$ independent over time

K maps stations to grid

2. $O(\cdot,t) = \mu_t \mathbf{1} + H\ O(\cdot,t\text{-}1) + G\ M(\cdot,t) + n(\cdot,t)$

$n(\cdot,t)$ independent over time

& e's and n's independent

$\boxed{\text{Keys}}$ Try <u>simple</u> Covariances for errors.:

1. Independent measurement errors $e(\cdot,t)$
2. Some spatial dependence explained by conditioning on $O(\cdot,t\text{-}1)$ & $M(\cdot,t)$
3. Parameterized H: current location and it's 4 nearest neighbors

(Requires a "boundary process")

4. Parameterized G: regress onto Met.

$M(\cdot,t)$ vrs $M(\cdot,t\text{-}1)$

We don't predict Met: leave it to experts:

In predictive mode, their forecasts!!

5. Model $\mu_{\mathbf{t}}$

Time series: $[\mu_{\mathbf{1}}, \ldots, \mu_{\mathbf{T}} \mid$ M, $\boldsymbol{\theta}]$

$=[\mu_{\mathbf{1}}|\mathbf{M}]$ product $[\mu_{\mathbf{t}}|\mu_{\mathbf{t-1}}$, M, $\boldsymbol{\theta}]$

Using <u>mixture model</u>:

A. “Normal pressure regime”

Simple autoregression

B. “High pressure regime”

Simple autoregression, but larger intercept

Regionally averaged Pressure through

Day t-1 used to randomly pick regime

(Probit model)

$\boxed{\text{Discussion}}$

A. Skeletons in the closet:

*** Computation (MCMC).

*** [$\boldsymbol{\theta}$] hard, but important.

B. Hierarchical vision:

*** Extend to [PM, O]=[PM | O][O]

*** Link to "Regional" via models:

***** [O(region)|O(urban)]

***** Use Dispersion models &

***** weather prediction

*** HBM: results plus uncertainty

Some References on HBM:

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