UNITED STATES ENVIRONMENTAL PROTECTION AGENCY
Office of Air Quality Planning and Standards (OAQPS)
Research Triangle Park, North Carolina 27711

# MEMORANDUM

**SUBJECT:** A Comparison between Different Rollback Methodologies Applied to Ambient Ozone Concentrations

**FROM:** Michael Rizzo, EPA-OAQPS, Air Quality Data Analysis Group

**TO:** Ozone NAAQS Review Docket (OAR-2005-0172)

**DATE:** November 7, 2005

For the prior ozone NAAQS review, one of the methods, referred to as the "Quadratic Method," developed for adjusting ozone ambient air concentrations to simulate just meeting alternative standards combined both linear and quadratic elements to reduce larger concentrations more than smaller ones.[1] In this regard, the Quadratic method attempts to account for reductions in emissions without greatly affecting lower concentrations near ambient background levels. Other rollback algorithms have either fit the data to a particular distribution such as the Weibull method or used a linear, proportional rollback where all of the ambient measurements are reduced equally regardless of their individual magnitudes.[2]

This memorandum will compare two of the above mentioned rollback methodologies: quadratic and percentile proportional. As the name implies, the quadratic method uses a quadratic equation to reduce higher ozone concentrations more than smaller ones. The amount of rollback depends on the magnitude of the reduction of the existing fourth maximum to meet the standard. Sites which have data with high ozone concentrations are subjected to a more substantial rollback than those which are at or below the National Ambient Air Quality Standards. In contrast, the percentile proportional rollback uses a dual linear approach where ozone concentrations less than a specified percentile are not rolled back while those greater than the percentile value are proportionally rolled back based upon the difference between the measured fourth maximum and the calculated value needed to attain the standard.

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1 Duff, Marcus; Horst, Robert L.; Johnson, Ted R.; "Quadratic Rollback: A Technique to Model Ambient Concentrations Due to Undefined Emission Controls". Paper No. 98-TA32.07 Air and Waste Management Annual Meeting. San Diego, California. June 14-18, 1998.

2 Johnson, Ted; A Guide to Selected Algorithms, Distributions, and Databases Used in Exposure Models Developed by the Office of Air Quality Planning and Standards. EPA Grant No. CR827033. May 2002. http://www.epa.gov/ttn/fera/data/human/report052202.pdf

**Methodology**

*Quadratic Rollback*

The Quadratic Rollback method takes the form of the following equation

$$C'_j = r_j C_j$$

where : $C'_j$ is the rolled back concentration
$r_j$ is the quadratic rollback factor unique to each measurement
$C_j$ is the original measured concentration

The quadratic rollback factor is defined as

$$r_j = V - BC_j$$

where: V and B are positive constants determined from an individual site's measurements

In order to calculate the V and B constants, other parameters must be known first. These are listed below:

$$I_i = \sum_{j \in i} C_j / N_i$$

where: $I_i$ is the average concentration for time period i which, in the case of the current ozone standard, refers to an 8 hour time period so $I_i$ is an 8 hour average
$C_j$ is the original measured concentration
$N_i$ is the number of hours for time period i

$$J_i = \max(C_j)_{j \in i}$$

where: $J_i$ is the maximum one hour value for time period i which, in this case, is the maximum 1 hour value in the 8 hour time period

$$Q_i = \sum_{j \in i} C_j^2 / N_i$$

where: $Q_i$ is the average of the squared concentrations for time period i

$$J = \max(J_i)$$

where: J is the maximum of all time periods, i=1, 2, 3, ….. etc. and is also the maximum value over the entire length of time analyzed

For each time period i, a transformation factor, $X_i$, is computed using the above parameters:

$$X_i = 2JI_i - Q_i$$

The appropriate $X_i$ which corresponds to the metric corresponding the standard that the data being rolled back to is compared to two times the product of the maximum one hour value (J) and a standard concentration level (S). For example, if the current ozone standard is being examined, the fourth highest $X_i$ from all of the time periods is used. The standard concentration value can be the actual value to meet the NAAQS such as 0.084 ppm for the 8 hour ozone standard. In this case, S refers to the target value of the $4^{th}$ maximum 8 hour average for 2004 whose calculation is described below. A metric is calculated of the following form to determine the final equation used for the rollback calculation:

$$V = 2JS / X$$

The B coefficient is calculated for each concentration for time period i as:

$$B_i = (I_i - S) / Q_i$$

The appropriate $B_i$ is chosen in the same manner as the appropriate $X_i$ described above. For example, if the metric for the standard is the $4^{th}$ highest 8 hour value, the $B_i$ corresponding to the $4^{th}$ highest 8 hour value is used in the rollback calculation.

Thus, V is used as the metric to determine which equation is used for the final rollback calculation. If V is greater than or equal to 1 then this is considered to be the pure quadratic case and the V coefficient maintains a value of 1 for all time periods. The rollback equation then becomes:

$$C_j' = C_j - B(C_j^2)$$

where: B has the value of $B_i$ corresponding to the standard's metric being tested

If V is between 0 and 1, this is the mixed linear-quadratic case. The rollback equation then becomes:

$$C_j' = VC_j - \{(VI_m - S)/Q_m\}C_j^2$$

where: $I_m$ and $Q_m$ refer to the I and Q quantities for period i which refers to the metric of the standard

*Data*

Data from eight sites within three major urban areas were used to calculate rolled back ozone concentrations using the quadratic method. For each site, a high and low ozone year was chosen based on historical data. The information detailing the high and low years and the corresponding eight hour averaged 4$^{th}$ maxima are provided in Table I.

Table I: List of Sites and Respective High/Low Ozone Years Utilized

| Site ID | High ozone | | Low ozone | | City |
|---|---|---|---|---|---|
| | Year | 8 hour 4th max | Year | 8 hour 4th max | |
| 060371601 | 1994 | 127 | 2002 | 74 | Los Angeles |
| 060372005 | 1994 | 132 | 2001 | 90 | Los Angeles |
| 060658001 | 1994 | 148 | 2000 | 106 | Los Angeles |
| 060711004 | 1994 | 148 | 2002 | 105 | Los Angeles |
| 171630010 | 1995 | 84 | 2001 | 78 | St. Louis |
| 291831002 | 1995 | 112 | 2001 | 85 | St. Louis |
| 420170012 | 1995 | 111 | 2003 | 87 | Philadelphia |
| 420450002 | 1995 | 108 | 2003 | 80 | Philadelphia |

The ozone concentrations measured in the high year were rolled back to the low year concentrations based on the differences in the 4$^{th}$ maxima.

The value to roll the fourth maximum 8 hour ozone values back to for each monitor-year is denoted as S. The value of S for each monitor-year is determined by the amount of rollback required to have the average fourth maximum ozone concentration over three years attain the standard. To accomplish this, the design value for each site is multiplied by a reduction factor calculated as:

$$\text{Reduction Factor} = C_{att}/C_{dv}*100$$

where: $C_{att}$ is the attainment concentration for the ozone standard which is 0.084 ppm

$C_{dv}$ is the average of the fourth maxima at the design value monitor for a particular area over a three year period

The value (S) which each fourth maximum in a three year is rolled back to is calculated as:

$$S_i = \text{Reduction Factor} * C_{max\,i}$$

where: $S_i$ is the rollback value for year i

$C_{max\,i}$ is the fourth maximum for year i

For the purpose of this work, S reflects the low year 4$^{th}$ maximum concentration to which the high year's 4$^{th}$ maximum concentration needed to be rolled back.

*Percentile Proportional Rollback*

The percentile proportional rollback technique is a linear approach where the relationship between the adjusted and actual hourly ozone concentrations changes at a specified value represented by a percentile of the hourly data's distribution. The first of the two relationships follows a line with slope equal to one between the adjusted and actual ozone concentrations up to the percentile value chosen. This can be represented as:

$$C_{adj} = C_{act} \text{ when } C_{act} <= C_{pctl}$$

where: $C_{adj}$ is the adjusted hourly ozone concentration
$C_{act}$ is the actual hourly ozone concentration
$C_{pctl}$ is the value of the chosen percentile (e.g. 50$^{th}$, 60$^{th}$, 70$^{th}$, 80$^{th}$ and 90$^{th}$) of the hourly ozone data distribution

The second relationship is a line which passes through the points at the chosen percentile value and the values of the adjusted and actual air quality indicator for the design value site. This allows for a continuous relationship across the range of hourly ozone concentrations with two segments representing the values below and above the chosen percentile. This relationship can be expressed as:

$$C_{adj} = \beta_0 + \beta_1 * C_{act} \text{ when } C_{act} > C_{pctl}$$

where: $C_{adj}$ is the adjusted hourly ozone concentration

$C_{act}$ is the actual hourly ozone concentration
$\beta_0$ is the intercept of the line

$\beta_1$ is the slope of the line

$C_{pctl}$ is the value of the chosen percentile (e.g. 50$^{th}$, 60$^{th}$, 70$^{th}$, 80$^{th}$ and 90$^{th}$) of the hourly ozone data distribution

The slope ($\beta_1$) and intercept ($\beta_0$) of the line are calculated from the two points which the line passes through.

$$\beta_1 = (S-C_{pctl})/(AQI-C_{pctl})$$

$$\beta_0 = C_{pctl} - (\beta_1 * C_{pctl})$$

where: AQI is the air quality indicator which in this case is the 8 hour 4$^{th}$ maximum value

S is the concentration which the high year 8 hour 4$^{th}$ maximum concentration is rolled back to

The percentile proportional rollback was calculated using the 50$^{th}$, 60$^{th}$, 70$^{th}$, 80$^{th}$ and 90$^{th}$ percentiles from the hourly ozone concentrations from the sites listed in Table I. Rolled back values were truncated after the third decimal place to be consistent with the current ozone reporting methodology as well as the procedures for calculating 8 hour ozone concentrations.

**Results and Discussion**

Table II in the Appendix displays the differences between the quadratic rollback and percentile proportional rollback methods' distributions to the concentration distribution of the low ozone year. Five percentiles ranging from the 50$^{th}$ to the 90$^{th}$ were used to calculate the proportional rollback. The various figures comparing the two methods to the low year's distribution show that in some cases there is little or no difference. This usually occurs at sites where the rollback's magnitude is not high such as site 171630010 in St. Louis. However, larger differences occur when the ozone concentrations have to be rolled back by more than several parts per billion. When this occurs, the percentile proportional rollback differs with the quadratic rollback more at larger concentrations and is dependent on the percentile chosen for the proportional method to differentiate between low and high ozone concentrations.

When compared to the low year distribution, the percentile proportional rollback appears to differ more at specific percentile cut points than does the quadratic which does not rely on any specified percentile to dictate the amount of rollback required. This makes the quadratic method easier to implement since it is not necessary to iterate through a series of percentiles to find the best distributional fit with the original data. Furthermore, the quadratic method does not exhibit the small spike in the number of rolled back ozone concentrations at certain intervals as the percentile proportional rollback does. This is most likely due to the change in slope at a specified percentile.

**APPENDIX**

Table II.  Comparison between the Distributions for the Quadratic Rollback, the Percentile Proportional Rollback and the Low Year Ozone Concentrations

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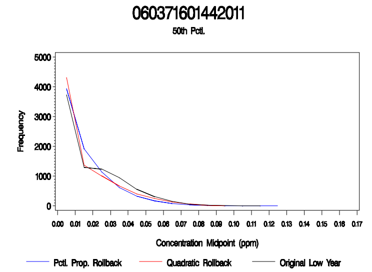
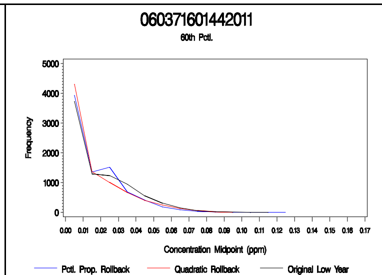
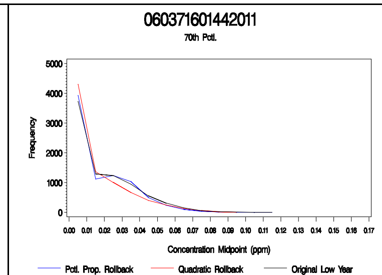
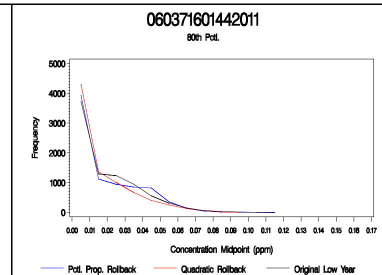
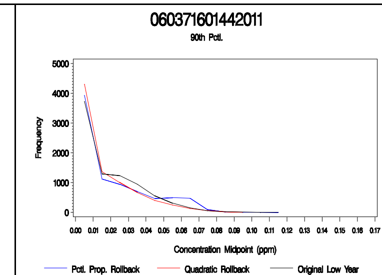
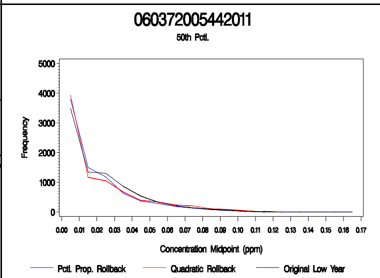
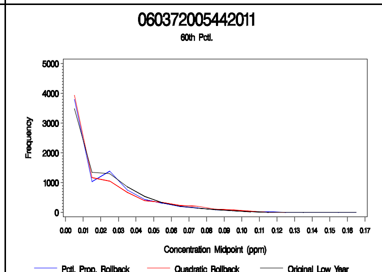
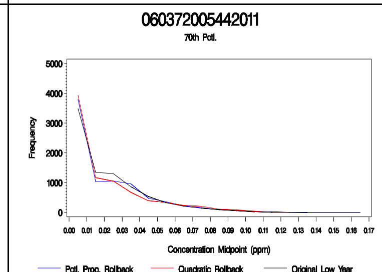
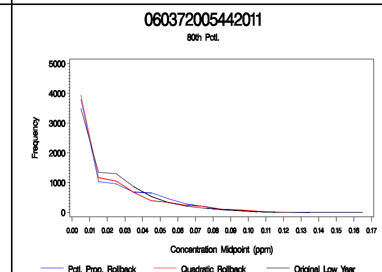
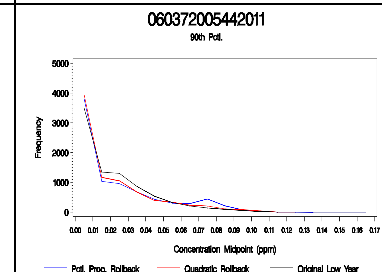
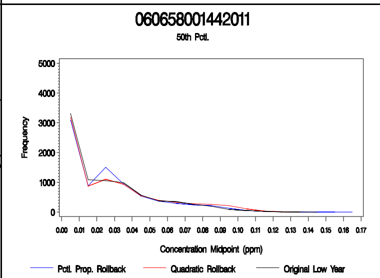
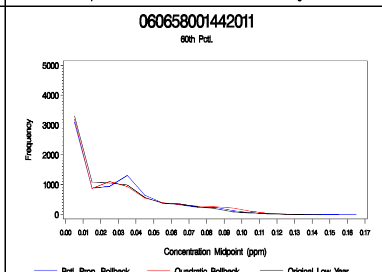
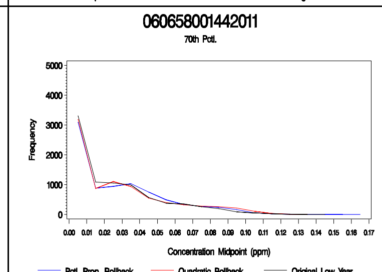
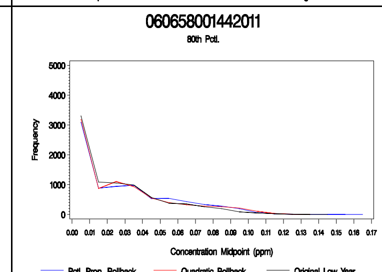
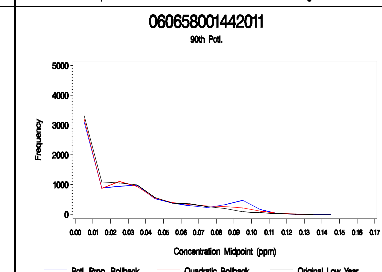
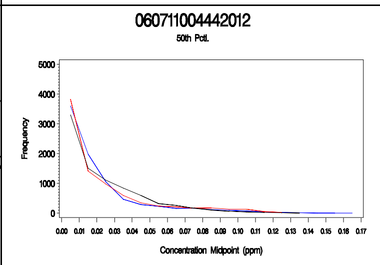
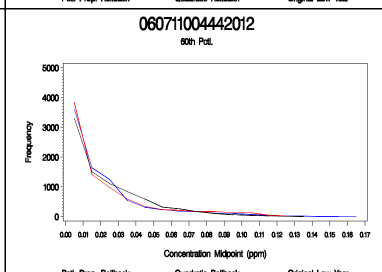
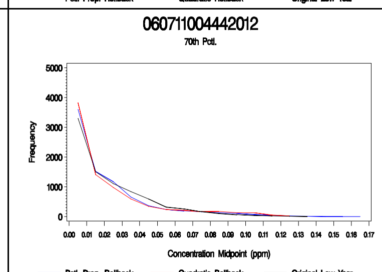
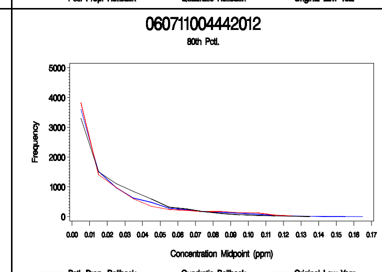
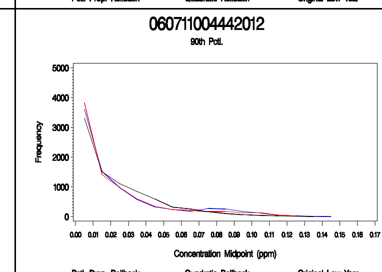
|---|---|---|---|---|---|
| Los Angeles, CA |  |  |  |  |  |
| Los Angeles, CA |  |  |  |  |  |
| Los Angeles, CA |  |  |  |  |  |
| Los Angeles, CA |  |  |  |  |  |

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